Stop on red, go on green . . .

"When you walk on a path going north, you will only meet people coming from the north. At the crossroads, you'll meet people coming from the east, from the west . . ."
—Nouk Bassomb

WHEN YOU'RE DRIVING down a road and you see a yellow light, don't you wonder when the light will turn red? Maybe there should be an additional light—say, a blue one—that tells you that the yellow light will be changing to red any moment now. But then again, maybe there should be an orange light that tells you that the blue light will be ending soon and that the red light is imminent. But then again . . .

Who needs yellow lights at intersections? Who decides whether the yellow light should be one second, two seconds, or four seconds? Are yellow lights always set to encourage safe driving? Let's analyze what happens when you approach a yellow light.

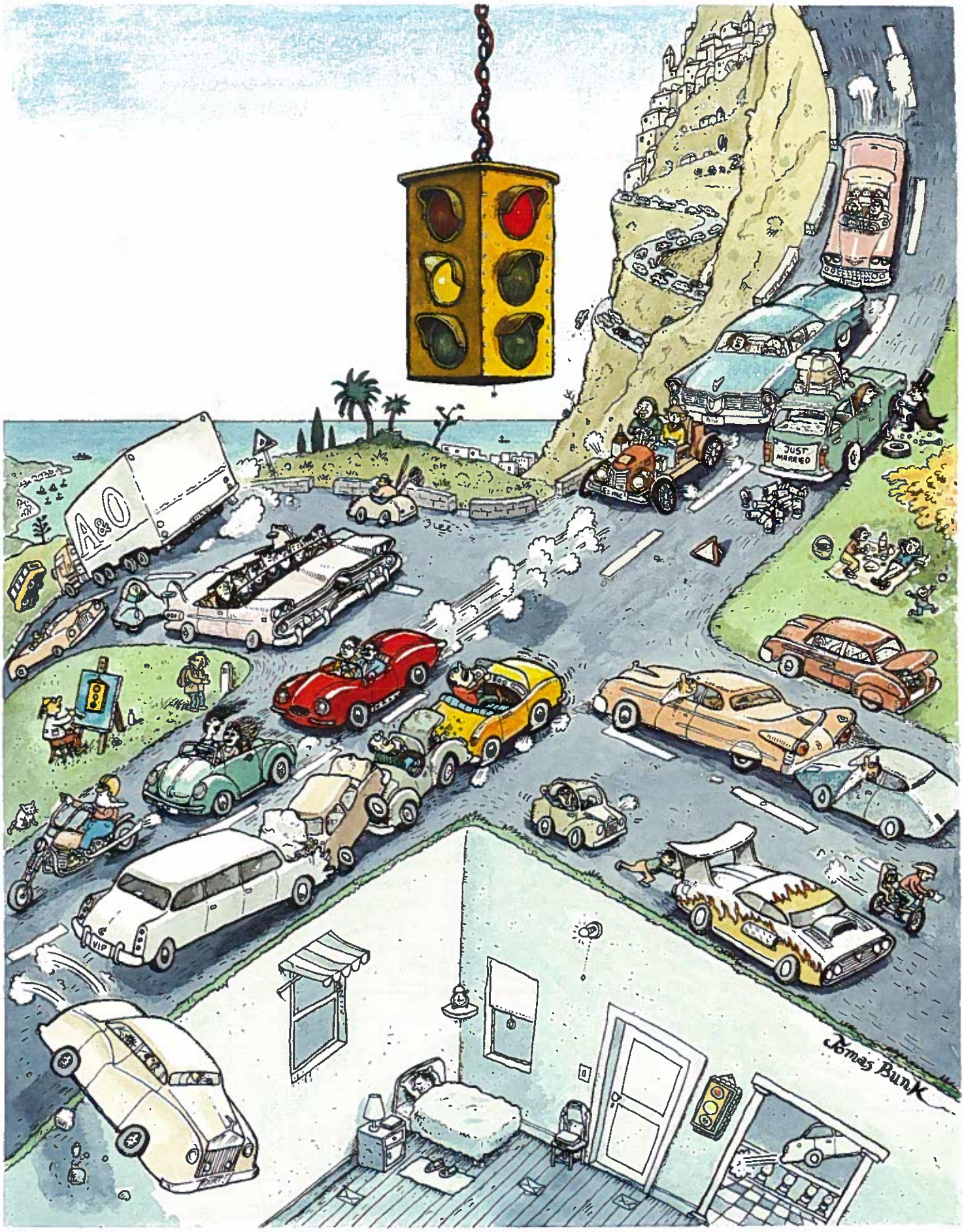
As you drive down the road at a certain speed, you may see the light turn from green to yellow. You must make a decision to keep going or to step on the brakes and come to a stop. If you're relatively close to the intersection, you know that you can continue at the same speed and make it through while the light is still yellow. If your distance to the intersection is larger, you may decide to stop.

Let's assume that you want to keep going. To calculate your safe distance from the intersection, we simply calculate the distance you must go to get through the intersection while the light is yellow. This may be easier to follow if we use some real numbers as an example. Let's assume that the speed limit is 50 km/h, which is equivalent to 30 mph or 23 m/s. Let's also assume that the yellow light is on for 3.0 s before the light turns red. Therefore, you must travel a distance of 69 m during the time the light is yellow. If the width of the intersection is 15 m, you can safely proceed through the intersection if you are closer than 54 m. We'll call this the "go zone."

If you decide to stop when you see

Crossroads are always a risky challenge, with or without a traffic light. Decisions have to be made quickly, sometimes there is not enough time to take all the possibilities into consideration. We can stop suddenly and have all the cars behind us bumping into each other, or try to speed up and get stuck in the middle of the intersection. We can take a wrong turn and end up off the road. But not only as drivers but as humans in general standing at a crossroads, we sometimes have to make tough decisions not knowing where the turn will lead us, to disaster or success, to glory or tragedy. Not being able to see the future, we may decide not to do anything but to go to sleep and dream about flying cars that can go anywhere anytime. But watch out—there is already a Rolls Royce slipping off the road and on its way to crashing right in the middle of your dreams.

—T.B.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the light turn yellow, you must know the distance you will travel as you move your foot from the gas pedal to the brake (the coasting distance) and the distance it takes your car to stop (the braking distance). Once again, let's look at some real numbers and perform a calculation. The car is once again traveling at 23 m/s. If your response time is 1.0 s, the car will travel a distance of 23 m. If the deceleration of the car is 5 m/s^2, the car will travel an additional 53 m while braking. This distance is calculated according to the following equation:

$$v_f^2 - v_0^2 = 2as,$$

where v_f is the final velocity, v_0 is the initial velocity, a is the acceleration, and s is the distance traveled. The car can be safely stopped if it is at least 76 m from the intersection. We'll call this the "stop zone."

But wait—what happens if you're 65 m from the intersection? If you try to stop, you'll find yourself in the intersection. If you try to continue, you'll find yourself going through a red light. You're in trouble! We'll call this the "dilemma zone."

A safer intersection would not have a dilemma zone. If the yellow light time were 4.0 s, the go zone would be 77 m. The stop zone would still be 76 m. If you are closer than 77 m, you can safely proceed. If you are farther than 76 m, you can safely stop. If you are between 76 and 77 m, you can safely go or stop. This "overlap zone" provides for a safe intersection.

Rather than using data from a single intersection, our problem asks you to do the work of a highway engineer and provide the relevant equations for safe intersections.

A. What is the general equation for the (a) go zone, (b) stop zone, (c) dilemma zone or overlap zone?

Assume a response time t_r, a maximum braking acceleration a, a yellow light time t_y, a speed v_0, an intersection width w, and a car length l.

B. For what speeds will there always be a dilemma zone?

C. Rewrite the equations in part A assuming that the car is going downhill when you see the yellow light.

Solution